Power System State Estimation and Optimal Measurement Placement For Distributed Multi-Utility Operation

Final Project Report

Project Team

Ali Abur
Garng M. Huang
Texas A&M University

PSERC Publication XX-XX

November 2002
Information about this Project

For information about this project contact:

A. Abur  
Professor  
Texas A&M University  
Department of Electrical Engineering  
College Station, TX 77843-3128  
Phone: 979 845 1493  
Fax: 979 845 9887  
Email: abur@tamu.edu

Power Systems Engineering Research Center

This is a project report from the Power Systems Engineering Research Center (PSERC). PSERC is a multi-university Center conducting research on challenges facing a restructuring electric power industry and educating the next generation of power engineers. More information about PSERC can be found at the Center’s website: http://www.pserc.wisc.edu.

For additional information, contact:

Power Systems Engineering Research Center  
Cornell University  
428 Phillips Hall  
Ithaca, New York 14853  
Phone: 607-255-5601  
Fax: 607-255-8871

Notice Concerning Copyright Material

Permission to copy without fee all or part of this publication is granted if appropriate attribution is given to this document as the source material. This report is available for downloading from the PSERC website.

© 2002 Texas A&M University. All rights reserved.
ACKNOWLEDGEMENTS

The work described in this report was sponsored by the Power Systems Engineering Research Center (PSERC). We express our appreciation for the support provided by PSERC’s industrial members and by the National Science Foundation under grant NSF EEC 0001880 received under the Industry / University Cooperative Research Center program.

The industry advisors for the project were Mani Subramanian, ABB Network Management; Don Sevcik, Center Point Energy; Bruce Dietzman, Oncor. Their suggestions and contributions to the work are appreciated.
EXECUTIVE SUMMARY

The new power markets induce changes in the way the transmission grid is operated and as a result, an increased number of power transactions take place creating unusual power flows through the system. Monitoring these flows reliably and accurately requires a robust measurement system. Furthermore, unlike the conventional systems, modern power systems are equipped with advanced power flow controllers or flexible A.C. transmission system (FACTS) devices for redirecting power flows to handle congestion. Monitoring these devices and their parameters is also becoming important. Finally, existence of several inter-utility power exchanges presents a need for addressing the multi-utility data exchange issues in the new power market environment.

This project is concerned about the above summarized issues. Specifically, a systematic method of placing meters either to upgrade and existing measurement system or to build one from scratch, is developed. This method not only ensures observability of the system for a base case operating topology, but also accounts for expected contingencies and measurement losses. In order to address the issue of FACTS devices, a new estimator is developed. This estimator is capable of incorporating the power flow controllers along with their operating and parameter limits, into the state estimation formulation. Finally, a new measurement exchange strategy for multi-utility distributed operation is developed and illustrated to improve the overall measurement reliability of the system.

The developed methods of this project are implemented in form of prototype software and simulations are carried out on test systems.
Table of contents

Part I: State Estimation of Power Systems Embedded with FACTS Devices .............. 1

I. INTRODUCTION ........................................................................................................ 1
   1.1 Introduction................................................................. 1
   1.2 Problem Statement ....................................................... 1

II. PROPOSED ALGORITHM................................................................. 3
   2.1 Steady state model of UPFC .............................................. 3
   2.2 HATCHTEL’s augmented matrix method [3,4] .......................... 4
   2.3 Observability Analysis .................................................... 6
   2.4 Equations........................................................................... 7
   2.5 Algorithm......................................................................... 9

III. NUMERICAL EXAMPLES .............................................................. 10
   3.1 14-bus system............................................................. 10
   3.2 30-bus system............................................................. 15
   3.3 Conclusion....................................................................... 21

REFERENCES ......................................................................................... 21

Part II: Optimal Meter Placement for Measurements loss and Branch Outages........... 23

I. INTRODUCTION ................................................................................... 23
   1.1 Introduction......................................................................... 23
   1.2 Problem Statement ....................................................... 24

II. PROPOSED ALGORITHM........................................................................ 25
   2.1 H matrix .............................................................................. 25
   2.2 Candidate measurements identification .............................. 26
   2.3 Optimal Meter Placement ............................................... 29
   2.4 Algorithm........................................................................... 31

III. NUMERICAL EXAMPLES .............................................................. 32
   2.1 6-bus system....................................................................... 32
   2.2 14-bus system..................................................................... 35
   2.3 30-bus system..................................................................... 38
   2.4 57-bus system..................................................................... 42
   2.5 Conclusions....................................................................... 46

REFERENCES ......................................................................................... 47
PART I:  STATE ESTIMATION OF POWER SYSTEMS EMBEDDED WITH FACTS DEVICES

I. INTRODUCTION

1.1 Introduction

After the establishment of power markets with transmission open access, the significance and use of FACTS devices for manipulating line power flows to relieve congestion and optimize the overall grid operation have increased. As a result, there is a need to integrate the FACTS device models into the existing power system applications. This report will present an algorithm for state estimation of network embedded with FACTS devices. Furthermore, it will be shown via case studies that the same estimation program can also be used for determine the controller setting for a desired operation condition.

There are several kinds of FACTS devices. Thyristor-switched series capacitors (TCSC) and thyristor switched phase shifting transformer (TCPST) can exert a voltage in series with the line and therefore can control the active power through a transmission line [3]. On the other hand, the Unified Power Flow Controller (UPFC) has a series voltage source and a shunt voltage source, allowing independent control of the voltage magnitude, the real and reactive power flows along a given transmission line [1,2]. In this report, only one device namely the UPFC will be considered due to its complexity and versatility in controlling the power flows.

1.2 Problem Statement

Flexible A.C. transmission systems [FACTS] are more and more used in large power systems for their significance of manipulating line power flows. Traditional state estimation problems without integrating FACTS devices will not be suitable for power
systems embedded with FACTS. By introducing FACTS models into state estimation problems, we will introduce complexity, but also flexibility into this program.

State estimation in power system can be formulated as a nonlinear weighted least square (WLS) problem. It has a set of measurement equations: $z = h(x) + \varepsilon$; a set of equality constraints $c(x) = 0$, representing the zero injections of buses and the zero active power exchange between the power system and FACTS devices; a set of inequality constraints $f(x) \leq s$, representing the Var limits on generators, transformer tap ratio limits and the power and voltage limit of FACTS devices.

This paper will present an algorithm to solve this nonlinear weighted least square problem. By solving the problem we can not only estimate the state variables (bus voltages and phase angles) of power system bus can also determine the controller settings of FACTS devices for a desired operating condition.

In this report, an approach that incorporates FACTS devices into the state estimation will be presented. First, a steady-state model of the UPFC [1,2] with operating and parameters limits will be introduced. Then, the commonly used Hatchtel’s augmented matrix method [3,4] will be used to implement a numerically robust and computationally efficient state estimator, which is also flexible enough to account for various device constraints. To treat the inequality constraints, we will introduce the Logarithmic barrier function method [5] and integrate it into Hachtel’s matrix. Simulation results for typical systems are shown at the end of part one. It will be shown via case studies that this program can also be used for determining the controller settings of UPFC for a desired operating condition.
II. PROPOSED ALGORITHM

2.1 Steady state model of UPFC

The Unified Power Flow Controller (UPFC) [1,2] can control the voltage magnitude, real and reactive power flows simultaneously. The real physical model of UPFC consists of two switching converters as illustrated in Figure 1.1. These inverters are operated from a common dc link provided by a dc storage capacitor. This arrangement functions as an ideal ac to ac power converter in which the real power can freely flow in either direction between the ac terminals of the two inverters and each inverter can independently generate (or absorb) reactive power at its own ac output terminal [2].

![Fig. 1.1. Basic circuit arrangement of the Unified Power Flow Controller](image1)

The steady state model of UPFC consists of two ideal voltage sources, one in series and one in parallel with the associated line, as shown in Figure 1.2. Neglecting UPFC losses, during steady-state operation it neither absorbs nor injects real power into the system [2].

![Fig. 1.2. Steady state model of UPFC](image2)
The constraint $P_B + P_E = 0$ in Figure 1.2 implies that:

- No real-power is exchanged between the UPFC and the system.
- The two sources are mutually dependent.

The real and reactive power going through line k-m can be formulated by equations (1.1) to (1.4).

\[
P_{km} = \frac{|V_k| |V_m|}{X_B} \sin \theta_{km} + \frac{|V_k| |V_E|}{X_E} \sin \theta_{k,E} - \frac{|V_k| |V_B|}{X_B} \sin \theta_{k,B} \tag{1.1}
\]

\[
Q_{km} = \frac{X_E + X_B}{X_B X_E} |V_k|^2 - \frac{|V_k| |V_m|}{X_B} \cos \theta_{km} - \frac{|V_k| |V_E|}{X_E} \cos \theta_{k,E} + \frac{|V_k| |V_B|}{X_B} \cos \theta_{k,B} \tag{1.2}
\]

\[
P_{mk} = \frac{|V_k|^2}{X_B} \sin \theta_{mk} + \frac{|V_m| |V_B|}{X_B} \sin \theta_{m,B} \tag{1.3}
\]

\[
Q_{mk} = \frac{|V_m|^2}{X_B} - \frac{|V_k| |V_m|}{X_B} \cos \theta_{mk} - \frac{|V_m| |V_B|}{X_B} \cos \theta_{m,B} \tag{1.4}
\]

Variables $V_E, \theta_E, V_B, \theta_B$ are the control parameters of UPFC. There are equality and inequality constraints of UPFC, which can be formulated by equations (1.5) to (1.9).

Real Power Constraints: $P_E + P_B = 0$ \hspace{1cm} (1.5)

Shunt Power Constraints: $\sqrt{P_E^2 + Q_E^2} \leq T_{E,max}$ \hspace{1cm} (1.6)

Series Power Constraints: $\sqrt{P_B^2 + Q_B^2} \leq T_{B,max}$ \hspace{1cm} (1.7)

Shunt Voltage Constraints: $|V_B| \leq V_{B,max}$ \hspace{1cm} (1.8)

Series Voltage Constraints: $|V_E| \leq V_{E,max}$ \hspace{1cm} (1.9)

2.2 HACHTEL’s augmented matrix method [3,4]

Power system state estimation problem can be formulated as a nonlinear least squares problem with a set of equality and inequality constraints [6].
\[
\begin{align*}
\text{Min} & \quad \frac{1}{2} r^T R^{-1} r \\
& \quad f(x) + s = 0 \\
& \quad c(x) = 0 \\
\text{s.t.} & \quad r - z + h(x) = 0 \\
& \quad s \geq 0
\end{align*}
\]

(1.10)

\[z = h(x) + r\] represents the equations for measurements, where \(z\) is the \((m \times 1)\) measurement vector, \(h(\cdot)\) is the \((m \times 1)\) vector of nonlinear functions, \(x\) is the \((n \times 1)\) state vector, \(r\) is the \((m \times 1)\) measurement error vector.

\(c(x) = 0\) represents the equality constraints, where \(c(\cdot)\) is the \((r \times 1)\) vector of nonlinear functions. These equality constraints represent the zero injection buses and the zero active power exchange between the system and FACTS devices.

\(f(x) \leq 0\) represents the inequality constraints, which represent the Var limits on generators, ratio limits of transformer tap and the power and voltage limits of the UPFC

\[\sqrt{P_E^2 + Q_E^2} \leq T_{E,\text{max}}, \quad \sqrt{P_B^2 + Q_B^2} \leq T_{B,\text{max}}, \quad \left| V_B \right| \leq V_{B,\text{max}}, \quad \left| V_E \right| \leq V_{E,\text{max}}.\]

\(s\) is a vector of slack variables used to convert the inequality constraints to equality constraints.

In order to solve the problem of (1.10), we will employ the interior point optimization method. In this method, the slack variable \(s\) are treated by appending a logarithmic barrier function to the objective function,

\[
\phi_\mu = \frac{1}{2} r^T R^{-1} r - \mu \sum_{k=1}^{p} \ln s_k
\]

(1.11)

where \(p\) is the number of inequality constraints and \(s_k\) is the kth element of the slack variable vector \(s\). The barrier parameter \(\mu > 0\) is forced to decrease towards zero as the iterations progress.
The Lagrangian function is given by (1.12).

\[
L_\mu = \frac{1}{2} \bar{r}^T R^{-1} \bar{r} - \mu \sum_{k=1}^{p} \ln s_k - \bar{\lambda}^T \left[ f(x) + s \right] - \rho^T g(x) - \pi^T \left[ r - z + h(x) \right]
\] (1.12)

By using the Kuhn-Karroush-Tucker (KKT) optimality conditions and replacing the nonlinear functions by their first order approximations, the solution to the nonlinear least squares problem will be obtained by iteratively solving the following linear equations:

\[
\begin{bmatrix}
D & 0 & 0 & F \\
0 & 0 & 0 & G \\
0 & 0 & R & H \\
F^T & G^T & H^T & 0
\end{bmatrix}
\begin{bmatrix}
\lambda \\
\rho \\
\pi \\
\Delta x
\end{bmatrix}
= \begin{bmatrix}
-f(x^k) \\
-g(x^k) \\
z - h(x^k) \\
0
\end{bmatrix}
\] (1.13)

The matrix on the left side will be referred as the K matrix. Matrices \( F, G, H \) are the gradient matrices of the functions \( f(x), g(x), h(x) \) respectively. \( D \) is built as (1.14), where \( S \) is the diagonal matrix whose \( k^{th} \) diagonal element is \( s_k \).

\[
D = \frac{1}{\mu} (S)^2
\] (1.14)

Solving (1.13) iteratively yields the solution for (1.10).

2.3 Observability Analysis

Observability analysis can be carried out using the numerical method. The Jacobian matrix \( \begin{bmatrix} F \\ G \\ H \end{bmatrix} \) will be decomposed into its lower and upper rectangular factors using the Peter-Wilkinson method. In case of zero pivots, pseudo measurements will be added to make the system observable. The pseudo measurements will indicate deficiencies
in the measurement system, both for the network states as well as FACTS device parameters.

2.4 Equations

This section provides the detailed equations for the measurements incident to a given line, both with and without a UPFC device.

2.15 Lines without an installed UPFC

Consider the two possible measurements 1 and 2 on line k-m.

Equations for meter 1 are:

\[ P_k = V_k^2 G_{kk} + V_k \sum_{j=1, j \neq k}^n V_j (G_{kj} \cos \theta_{kj} + B_{kj} \sin \theta_{kj}) \]  \hspace{1cm} (1.15)

\[ Q_k = -V_k^2 B_{kk} + V_k \sum_{j=1, j \neq k}^n V_j (G_{kj} \sin \theta_{kj} - B_{kj} \cos \theta_{kj}) \] \hspace{1cm} (1.16)

Equations for meter 2 are:

\[ P_k = -V_k^2 G_{km} + V_k V_m (G_{km} \cos \theta_{km} + B_{km} \sin \theta_{km}) \] \hspace{1cm} (1.17)

\[ Q_k = V_k^2 (B_{km} - C_{km}) + V_k V_m (G_{km} \sin \theta_{km} - B_{km} \cos \theta_{km}) \] \hspace{1cm} (1.18)
2.16 Lines controlled by a UPFC

Suppose a UPFC is installed on line k-m. Measurements 1, 2, 3, 4 are the measurements that can be placed on line k-m.

Equations for meter 1 are:

\[ P_k = V_k^2 (G_{kk} + G_{km}) + V_k \sum_{j=1}^{n} V_j (G_{kj} \cos \theta_{kj} + B_{kj} \sin \theta_{kj}) + V_k V_B B_B \sin \theta_{kB} - V_k V_k B_E \sin \theta_{kE} \]
\[ Q_k = -V_k^2 (B_{kk} + B_{km} + B_B + B_E) + V_k \sum_{j=1}^{n} V_j (G_{kj} \sin \theta_{kj} - B_{kj} \cos \theta_{kj}) - V_k V_B B_B \cos \theta_{kB} + V_k V_k B_E \cos \theta_{kE} \]

Equations for meter 2 are:

\[ P_{km} = V_k V_k B_B \sin \theta_{kB} + V_k V_B B_B \sin \theta_{kB} \]
\[ Q_{km} = V_k^2 B_B - V_k V_k B_B \cos \theta_{kB} \]

and,

\[ P_{km} = -V_k^2 G_{km} + V_k V_m (G_{km} \cos \theta_{km} + B_{km} \sin \theta_{km}) \]
\[ Q_{km} = V_k^2 (B_{km} - C_{km0}) + V_k V_m (G_{km} \sin \theta_{km} - B_{km} \cos \theta_{km}) \]

Equations for meter 3 are:

\[ P_{mk} = -V_m^2 G_m + V_m V_k (G_{mk} \cos \theta_{mk} + B_{mk} \sin \theta_{mk}) \]
\[ + V_m V_B (G_{km} \cos \theta_{mk} + B_{km} \sin \theta_{mb}) \]
\[ Q_{mk} = V_m^2 (B_{km} - C_{mk}) + V_m V_k (G_{kj} \sin \theta_{mkj} - B_{kj} \cos \theta_{mk}) + V_m V_B (G_{km} \sin \theta_{mb} - B_{km} \cos \theta_{mb}) \]  
\hfill (1.26)

Equations for meter 4 are:

\[ P_m = V_m^2 (G_{mn} + G_{mk} - G_{km}^*) + V_m \sum_{j=1}^{n} V_j (G_{mj} \cos \theta_{mj} + B_{mj} \sin \theta_{mj}) + V_m V_B (G_{km}^* \cos \theta_{mb} + B_{km}^* \sin \theta_{mb}) \]  
\hfill (1.27)

\[ Q_m = -V_m^2 (B_{nn} + B_{mk} - B_{km}^*) + V_m \sum_{j=1}^{n} V_j (G_{mj} \sin \theta_{mj} - B_{mj} \cos \theta_{mj}) + V_m V_B (G_{km}^* \sin \theta_{mb} - B_{km}^* \cos \theta_{mb}) \]  
\hfill (1.28)

2.5 Algorithm

The following algorithm is used in this program of state estimation. It is based upon the previously presented analysis and the reader is referred to the previous sections for the notation used in the following description of algorithm steps.

**Step 1:** Read network data and measurements;

**Step 2:** Initialize: \( x^{(k)} \), \( k = 0 \);

**Step 3:** Form \( K^{(k)} \) matrix;

**Step 4:** Calculate the equality and inequality constraints, measurements mismatch, and form the right hand side \( b^{(k)} \) vector. \( b^{(k)} = \begin{bmatrix} -f(x^k) \\ -g(x^k) \\ z - h(x^k) \\ 0 \end{bmatrix} \).

**Step 5:** Solve the equation: \( K^{(k)} \cdot \begin{bmatrix} \lambda \\ \rho \\ \pi \\ \Delta x \end{bmatrix} = b^{(k)} \), get \( \Delta x^{(k)} \).

**Step 6:** Update \( x \): \( x^{(k+1)} = x^{(k)} + \Delta x^{(k)} \).
**Step 7:** Terminate execution if \( \Delta x^{(k)} - \Delta x^{(k-1)} \leq \varepsilon \), and go to **step 8**, else, \( k = k+1 \) and go to **step 3**.

**Step 8:** Stop and print out results.

### III. NUMERICAL EXAMPLES

3.1 14-bus system

![IEEE-14 bus system diagram](image)

Fig. 1.5. IEEE-14 system

IEEE-14 bus system is shown in Figure 1.5. A FACTS device (UPFC) is installed on line 6-12, near bus 6. The parameters of UPFC are shown below.

Parameters of the installed UPFC device:

<table>
<thead>
<tr>
<th>From (bus)</th>
<th>To (bus)</th>
<th>( X_B )</th>
<th>( X_E )</th>
<th>( V_{B,max} )</th>
<th>( V_{E,max} )</th>
<th>( S_{B,max} )</th>
<th>( S_{E,max} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>12</td>
<td>0.7</td>
<td>0.7</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The developed program can be utilized in two different ways depending upon the purpose of study. It can be used as an estimator for the FACTS device parameters for a given set of measurements. The estimation will yield not only the system states but also the FACTS device parameters. It can also be utilized as a tool to estimate the
required values for the parameters of the FACTS devices in order to maintain a specific level of flow through a specified line. The amount of desired power flow through line 6-12, which happens to have a FACTS device installed on it, can be maintained by the use of this program and estimating the required settings of the control variables of this FACTS device.

First, the function of the program as an estimator will be illustrated.

Suppose that the system has voltage magnitude, bus injection and line flow measurement. The measurement values are shown below.

Voltage Measurements:

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.01870</td>
</tr>
<tr>
<td>14</td>
<td>1.03700</td>
</tr>
</tbody>
</table>

Bus Injection Measurements:

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>P</th>
<th>Q</th>
<th>Bus No.</th>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-0.94200</td>
<td>0.04393</td>
<td>5</td>
<td>-0.07600</td>
<td>0.01600</td>
</tr>
<tr>
<td>6</td>
<td>-0.11200</td>
<td>0.04718</td>
<td>7</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>8</td>
<td>0.00000</td>
<td>0.17357</td>
<td>9</td>
<td>-0.29500</td>
<td>-0.16600</td>
</tr>
<tr>
<td>10</td>
<td>-0.09000</td>
<td>-0.05800</td>
<td>11</td>
<td>-0.03500</td>
<td>-0.01800</td>
</tr>
<tr>
<td>13</td>
<td>-0.13500</td>
<td>-0.05800</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Line Flow Measurements:

<table>
<thead>
<tr>
<th>Bus No. (From)</th>
<th>Bus No. (To)</th>
<th>Active Power P</th>
<th>Reactive Power Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1.56771</td>
<td>-0.20378</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.73161</td>
<td>0.03568</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>0.27870</td>
<td>-0.09478</td>
</tr>
<tr>
<td>9</td>
<td>14</td>
<td>0.08801</td>
<td>0.03591</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
<td>0.13670</td>
<td>0.07187</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>0.12464</td>
<td>0.02372</td>
</tr>
</tbody>
</table>

The program is executed and all the unknown state and control variables of the UPFC device are estimated.
The state estimation results are shown below:

State variables:

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>V</th>
<th>θ (Degree)</th>
<th>Bus No.</th>
<th>V</th>
<th>θ (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.059985</td>
<td>0</td>
<td>2</td>
<td>1.044985</td>
<td>-4.979034</td>
</tr>
<tr>
<td>3</td>
<td>1.009984</td>
<td>-12.713291</td>
<td>4</td>
<td>1.018685</td>
<td>-10.318546</td>
</tr>
<tr>
<td>5</td>
<td>1.020265</td>
<td>-8.783118</td>
<td>6</td>
<td>1.069983</td>
<td>-14.261062</td>
</tr>
<tr>
<td>7</td>
<td>1.062103</td>
<td>-13.338571</td>
<td>8</td>
<td>1.089985</td>
<td>-13.338488</td>
</tr>
<tr>
<td>9</td>
<td>1.056632</td>
<td>-14.904211</td>
<td>10</td>
<td>1.051579</td>
<td>-15.076087</td>
</tr>
<tr>
<td>11</td>
<td>1.057220</td>
<td>-14.799891</td>
<td>12</td>
<td>1.067199</td>
<td>-14.363372</td>
</tr>
<tr>
<td>13</td>
<td>1.052852</td>
<td>-14.924974</td>
<td>14</td>
<td>1.037014</td>
<td>-15.909763</td>
</tr>
</tbody>
</table>

Voltages and powers of FACTS device:

<table>
<thead>
<tr>
<th>V̇_B</th>
<th>P_B</th>
<th>S_B</th>
<th>V̇_E</th>
<th>P_E</th>
<th>S_E</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1099∠60.0695°</td>
<td>0.0014</td>
<td>0.0128</td>
<td>1.0679∠-14.3099°</td>
<td>-0.0014</td>
<td>0.0035</td>
</tr>
</tbody>
</table>

Note that \( P_B + P_E = 0 \) and \( V_B \leq 1.0 \), \( S_B \leq 1.0 \), \( V_E \leq 1.0 \), \( S_E \leq 1.0 \), which correctly satisfy all the constraints.

The estimated and actual values for each measurement are given below.

<table>
<thead>
<tr>
<th>Bus Type</th>
<th>Bus No.1</th>
<th>Bus No.2</th>
<th>Real Value</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage meters</td>
<td>4</td>
<td></td>
<td>1.0187</td>
<td>1.0187</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td></td>
<td>1.0370</td>
<td>1.0370</td>
</tr>
<tr>
<td>Injection meters</td>
<td>11</td>
<td></td>
<td>-0.0350 – j 0.0180</td>
<td>-0.0350 – j 0.0180</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td></td>
<td>-0.0900 – j 0.0580</td>
<td>-0.0900 – j 0.0580</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td></td>
<td>-0.0760 – j 0.0160</td>
<td>-0.0760 – j 0.0160</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td></td>
<td>-0.001120 + j 0.0470</td>
<td>-0.001120 + j 0.0470</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td></td>
<td>-0.1350 – j 0.0580</td>
<td>-0.1350 – j 0.0580</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td></td>
<td>0.0000 + j 0.1725</td>
<td>0.0000 + j 0.1725</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td></td>
<td>0.0000 + j 0.0000</td>
<td>0.0000 + j 0.0000</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td></td>
<td>-0.2950 – j 0.1660</td>
<td>-0.2950 – j 0.1660</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td></td>
<td>-0.8420 + j 0.0435</td>
<td>-0.8420 + j 0.0435</td>
</tr>
</tbody>
</table>
Flow meters

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
<td>12</td>
<td>0.1246 + j 0.0237</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>1.5677 – j 0.2038</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>0.7316 + j 0.0357</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>7</td>
<td>0.2787 – j 0.0948</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>14</td>
<td>0.0880 + j 0.0359</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>13</td>
<td>0.1367 + j 0.0719</td>
</tr>
</tbody>
</table>

Next, the program’s usage as a power flow controller will be illustrated. Consider a case where the power flow data (with bus 1 chosen as slack with a voltage magnitude of 1.06) are given as below.

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>P</th>
<th>Q</th>
<th>Bus No.</th>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.18300</td>
<td>0.29695</td>
<td>3</td>
<td>-0.94200</td>
<td>0.04393</td>
</tr>
<tr>
<td>4</td>
<td>0.47800</td>
<td>0.03900</td>
<td>5</td>
<td>-0.07600</td>
<td>0.01600</td>
</tr>
<tr>
<td>6</td>
<td>-0.11200</td>
<td>0.04718</td>
<td>7</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>8</td>
<td>0.00000</td>
<td>0.17357</td>
<td>9</td>
<td>-0.29500</td>
<td>-0.16600</td>
</tr>
<tr>
<td>10</td>
<td>-0.09000</td>
<td>-0.05800</td>
<td>11</td>
<td>-0.03500</td>
<td>-0.01800</td>
</tr>
<tr>
<td>12</td>
<td>-0.06100</td>
<td>-0.01600</td>
<td>13</td>
<td>-0.13500</td>
<td>-0.05800</td>
</tr>
<tr>
<td>14</td>
<td>-0.14900</td>
<td>-0.05000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

First the state of the system with fixed UPFC parameters is estimated. The resulting system state and the power flow through the line 6-12 are given as:

State variables:

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>V</th>
<th>$\theta$ (Degree)</th>
<th>Bus No.</th>
<th>V</th>
<th>$\theta$ (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.060000</td>
<td>0</td>
<td>2</td>
<td>1.044993</td>
<td>-4.980830</td>
</tr>
<tr>
<td>3</td>
<td>1.009988</td>
<td>-12.717979</td>
<td>4</td>
<td>1.018608</td>
<td>-10.324100</td>
</tr>
<tr>
<td>5</td>
<td>1.020248</td>
<td>-8.782366</td>
<td>6</td>
<td>1.069953</td>
<td>-14.222568</td>
</tr>
<tr>
<td>7</td>
<td>1.061927</td>
<td>-13.368356</td>
<td>8</td>
<td>1.089978</td>
<td>-13.368356</td>
</tr>
<tr>
<td>9</td>
<td>1.056318</td>
<td>-14.946878</td>
<td>10</td>
<td>1.051296</td>
<td>-15.104578</td>
</tr>
<tr>
<td>11</td>
<td>1.057042</td>
<td>-14.795379</td>
<td>12</td>
<td>1.055177</td>
<td>-15.077467</td>
</tr>
<tr>
<td>13</td>
<td>1.050399</td>
<td>-15.159014</td>
<td>14</td>
<td>1.035760</td>
<td>-16.039228</td>
</tr>
</tbody>
</table>

Power flow in branch 6-12:
Then, the UPFC model is incorporated into the state estimation formulation. In this case, the system is underspecified and hence an extra equation is needed. This equation will be provided by the power flow measurement which will now be set equal to the desired value of the flow through the device in branch 6-12, which in this example is set equal to $0.1 + j0.1$, leaving all the other conditions the same.

The estimated state variables in this case are:

<table>
<thead>
<tr>
<th>Bus No. (From)</th>
<th>Bus No. (To)</th>
<th>$P_{6-12} + jQ_{6-12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>12</td>
<td>0.007780 + j0.02487</td>
</tr>
</tbody>
</table>

Control variables and estimated power of FACTS device are:

<table>
<thead>
<tr>
<th>$V_B$</th>
<th>$P_B$</th>
<th>$S_B$</th>
<th>$V_E$</th>
<th>$P_E$</th>
<th>$S_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.1236 \angle 9.0530^\circ$</td>
<td>0.0056</td>
<td>0.0159</td>
<td>$1.0000 \angle -14.6037^\circ$</td>
<td>-0.0056</td>
<td>0.0681</td>
</tr>
</tbody>
</table>

Note again that, $P_B + P_E = 0$, and $V_B \leq 1.0$, $S_B \leq 1.0$, $V_E = 1.0$, $S_E \leq 1.0$, which satisfy all the constraints.

Now the power flow in branch 6-12 is $0.1009 + j0.0999$, which closely match the desired set values, the slight difference being possibly due to the fact that the upper limit of $V_E$ is reached at the optimal solution.
This example illustrates that by setting the control variables of UPFC to $\dot{V}_B = 0.1236 \angle 9.0530^\circ$ and $\dot{V}_E = 1.0000 \angle -14.6037^\circ$, the power flow in branch 6-12 can be maintained at the desired amount.

3.2 30-bus system

IEEE-30 bus system is shown in Figure 1.6. A FACTS device (UPFC) is installed on line 4-6, near bus 6. The parameters of FACTS device are shown below.

Parameters of the installed UPFC device:

<table>
<thead>
<tr>
<th>From (bus)</th>
<th>To (bus)</th>
<th>$X_B$</th>
<th>$X_E$</th>
<th>$V_{B,\text{max}}$</th>
<th>$V_{E,\text{max}}$</th>
<th>$S_{B,\text{max}}$</th>
<th>$S_{E,\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>4</td>
<td>0.7</td>
<td>0.7</td>
<td>1.1</td>
<td>1.1</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
First, the function of the program as an estimator will be illustrated.

Suppose that the system has bus injection measurements and line flow measurements. The measurements values are shown as below. Bus 1 is the assumed slack bus with a specified voltage magnitude of 1.06.

Injection Meters:

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>P</th>
<th>Q</th>
<th>Bus No.</th>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.60137</td>
<td>-0.15420</td>
<td>2</td>
<td>0.18300</td>
<td>0.35540</td>
</tr>
<tr>
<td>3</td>
<td>-0.02400</td>
<td>-0.01200</td>
<td>5</td>
<td>-0.94200</td>
<td>0.17612</td>
</tr>
<tr>
<td>8</td>
<td>-0.30000</td>
<td>0.05226</td>
<td>9</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>10</td>
<td>-0.05800</td>
<td>-0.02000</td>
<td>13</td>
<td>0.00000</td>
<td>0.11163</td>
</tr>
<tr>
<td>12</td>
<td>-0.11200</td>
<td>0.11163</td>
<td>15</td>
<td>-0.08200</td>
<td>-0.02500</td>
</tr>
<tr>
<td>21</td>
<td>-0.17500</td>
<td>-0.11200</td>
<td>27</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>24</td>
<td>-0.08700</td>
<td>-0.06700</td>
<td>26</td>
<td>-0.03500</td>
<td>-0.02300</td>
</tr>
<tr>
<td>4</td>
<td>-0.07600</td>
<td>-0.01600</td>
<td>6</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

Flow Meters:

<table>
<thead>
<tr>
<th>Bus No. (From)</th>
<th>Bus No. (To)</th>
<th>Active Power P</th>
<th>Reactive Power Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1.68682</td>
<td>-0.20029</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0.91576</td>
<td>0.04609</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0.79259</td>
<td>0.02011</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>0.53422</td>
<td>0.01609</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>0.00000</td>
<td>-0.15412</td>
</tr>
<tr>
<td>13</td>
<td>12</td>
<td>0.00000</td>
<td>0.11163</td>
</tr>
<tr>
<td>12</td>
<td>16</td>
<td>0.05434</td>
<td>0.03611</td>
</tr>
<tr>
<td>14</td>
<td>15</td>
<td>0.01197</td>
<td>0.00809</td>
</tr>
<tr>
<td>16</td>
<td>17</td>
<td>0.01899</td>
<td>0.01736</td>
</tr>
<tr>
<td>15</td>
<td>18</td>
<td>0.05074</td>
<td>0.01838</td>
</tr>
<tr>
<td>18</td>
<td>19</td>
<td>0.01846</td>
<td>0.00879</td>
</tr>
<tr>
<td>10</td>
<td>21</td>
<td>0.16017</td>
<td>0.09975</td>
</tr>
<tr>
<td>15</td>
<td>23</td>
<td>0.04012</td>
<td>0.03217</td>
</tr>
<tr>
<td>22</td>
<td>24</td>
<td>0.06118</td>
<td>0.02992</td>
</tr>
<tr>
<td>25</td>
<td>26</td>
<td>0.03542</td>
<td>0.02364</td>
</tr>
<tr>
<td>25</td>
<td>27</td>
<td>-0.05405</td>
<td>-0.00118</td>
</tr>
</tbody>
</table>
The program is executed and all the unknown state and control variables of the UPFC device are estimated.

The state estimate results are shown below:

State variables:

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>V</th>
<th>θ (Degree)</th>
<th>Bus No.</th>
<th>V</th>
<th>θ (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.060000</td>
<td></td>
<td>2</td>
<td>1.043158</td>
<td>-5.278306</td>
</tr>
<tr>
<td>3</td>
<td>1.019352</td>
<td>-8.553746</td>
<td>4</td>
<td>1.010265</td>
<td>-10.311362</td>
</tr>
<tr>
<td>5</td>
<td>1.010131</td>
<td>-13.591051</td>
<td>6</td>
<td>1.012456</td>
<td>-10.248681</td>
</tr>
<tr>
<td>7</td>
<td>1.002980</td>
<td>-11.973942</td>
<td>8</td>
<td>1.011528</td>
<td>-10.986111</td>
</tr>
<tr>
<td>9</td>
<td>1.055265</td>
<td>-13.708757</td>
<td>10</td>
<td>1.051314</td>
<td>-15.510934</td>
</tr>
<tr>
<td>11</td>
<td>1.085641</td>
<td>-13.706705</td>
<td>12</td>
<td>1.063071</td>
<td>-15.517050</td>
</tr>
<tr>
<td>13</td>
<td>1.077737</td>
<td>-15.520280</td>
<td>14</td>
<td>1.050847</td>
<td>-16.441173</td>
</tr>
<tr>
<td>15</td>
<td>1.045770</td>
<td>-16.406170</td>
<td>16</td>
<td>1.051925</td>
<td>-15.917530</td>
</tr>
<tr>
<td>17</td>
<td>1.047809</td>
<td>-16.060072</td>
<td>18</td>
<td>1.037281</td>
<td>-16.906830</td>
</tr>
<tr>
<td>19</td>
<td>1.035051</td>
<td>-17.004135</td>
<td>20</td>
<td>1.032174</td>
<td>-16.056766</td>
</tr>
<tr>
<td>21</td>
<td>1.038988</td>
<td>-15.963820</td>
<td>22</td>
<td>1.039526</td>
<td>-15.955809</td>
</tr>
<tr>
<td>23</td>
<td>1.035933</td>
<td>-16.655654</td>
<td>24</td>
<td>1.027863</td>
<td>-16.395538</td>
</tr>
<tr>
<td>25</td>
<td>1.021634</td>
<td>-15.606883</td>
<td>26</td>
<td>1.003342</td>
<td>-15.943536</td>
</tr>
<tr>
<td>27</td>
<td>1.026629</td>
<td>-14.899834</td>
<td>28</td>
<td>1.008883</td>
<td>-10.891553</td>
</tr>
<tr>
<td>29</td>
<td>1.005645</td>
<td>-16.036445</td>
<td>30</td>
<td>0.994189</td>
<td>-16.915230</td>
</tr>
</tbody>
</table>

Voltages and power of FACTS device:

<table>
<thead>
<tr>
<th>$\bar{V}_B$</th>
<th>$P_B$</th>
<th>$S_B$</th>
<th>$\bar{V}_E$</th>
<th>$P_E$</th>
<th>$S_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.2089 \angle -93.6960^\circ$</td>
<td>0.0106</td>
<td>0.1797</td>
<td>$1.0077 \angle -10.3676^\circ$</td>
<td>-0.0106</td>
<td>0.0264</td>
</tr>
</tbody>
</table>
Note that $P_B + P_E = 0$ and $V_B \leq 1.1$, $S_B \leq 1.0$, $V_E \leq 1.1$, $S_E \leq 1.0$, which correctly satisfy all the constraints.

The estimated and actual values for each measurement are given below.

<table>
<thead>
<tr>
<th>Bus Type</th>
<th>Bus No.1</th>
<th>Bus No.2</th>
<th>Real Value</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.6014 – j 0.1542</td>
<td>2.6058 – j 0.1580</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.1830 + j 0.3554</td>
<td>0.1713 + j 0.3568</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.0240 – j 0.0120</td>
<td>-0.0155 – j 0.0189</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.9420 + j 0.1761</td>
<td>-0.9412 + j 0.1760</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-0.3000 + j 0.0523</td>
<td>-0.3007 + j 0.0523</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0 + j 0</td>
<td>0 + j 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-0.0580 – j 0.0200</td>
<td>-0.0580 – j 0.0200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0.0000 + j 0.1116</td>
<td>0.0005 + j 0.1104</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>-0.1120 – j 0.0750</td>
<td>-0.1111 – j 0.0776</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>-0.0820 – j 0.0250</td>
<td>-0.0816 – j 0.0277</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>-0.1750 – j 0.1120</td>
<td>-0.1755 – j 0.1113</td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>0 + j 0</td>
<td>-0.0000 – j 0.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>-0.0870 – j 0.0670</td>
<td>-0.0890 – j 0.0641</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>-0.0350 – j 0.0230</td>
<td>-0.0367 – j 0.0200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-0.0760 – j 0.0160</td>
<td>-0.0721 – j 0.0196</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0 + j 0</td>
<td>0 + j 0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Next the program’s usage as a power flow controller will be illustrated. Consider a case where the power flow data (where bus 1 is chosen as slack with a voltage magnitude of 1.06) are given as below.

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>P</th>
<th>Q</th>
<th>Bus No.</th>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.18300</td>
<td>0.31635</td>
<td>3</td>
<td>-0.024</td>
<td>-0.01200</td>
</tr>
<tr>
<td>4</td>
<td>-0.07600</td>
<td>-0.01600</td>
<td>5</td>
<td>-0.94200</td>
<td>0.16763</td>
</tr>
<tr>
<td>6</td>
<td>0.00000</td>
<td>0.00000</td>
<td>7</td>
<td>-0.22800</td>
<td>-0.10900</td>
</tr>
<tr>
<td>8</td>
<td>-0.30000</td>
<td>0.01710</td>
<td>9</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>10</td>
<td>-0.05800</td>
<td>-0.02000</td>
<td>11</td>
<td>0.00000</td>
<td>0.24000</td>
</tr>
<tr>
<td>12</td>
<td>-0.11200</td>
<td>-0.07500</td>
<td>13</td>
<td>0.00000</td>
<td>0.24000</td>
</tr>
<tr>
<td>14</td>
<td>-0.06200</td>
<td>-0.01600</td>
<td>15</td>
<td>-0.08200</td>
<td>-0.02500</td>
</tr>
<tr>
<td>16</td>
<td>-0.03500</td>
<td>-0.01800</td>
<td>17</td>
<td>-0.09000</td>
<td>-0.05800</td>
</tr>
</tbody>
</table>
First, the state of the system with fixed UPFC parameters is estimated. The estimated system state and the power flow through line 6-4 are shown below:

State variables:

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>V</th>
<th>θ (Degree)</th>
<th>Bus No.</th>
<th>V</th>
<th>θ (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.060000</td>
<td>0.000000</td>
<td>2</td>
<td>1.031817</td>
<td>-5.659963</td>
</tr>
<tr>
<td>3</td>
<td>1.007851</td>
<td>-8.391450</td>
<td>4</td>
<td>0.996201</td>
<td>-10.146324</td>
</tr>
<tr>
<td>5</td>
<td>0.990674</td>
<td>-14.968341</td>
<td>6</td>
<td>0.990340</td>
<td>-11.947724</td>
</tr>
<tr>
<td>7</td>
<td>0.981609</td>
<td>-13.742166</td>
<td>8</td>
<td>0.986904</td>
<td>-12.705754</td>
</tr>
<tr>
<td>9</td>
<td>1.033517</td>
<td>-15.428855</td>
<td>10</td>
<td>1.022804</td>
<td>-17.205907</td>
</tr>
<tr>
<td>11</td>
<td>1.077471</td>
<td>-15.509198</td>
<td>12</td>
<td>1.047093</td>
<td>-16.475904</td>
</tr>
<tr>
<td>13</td>
<td>1.077845</td>
<td>-16.476342</td>
<td>14</td>
<td>1.028016</td>
<td>-17.406134</td>
</tr>
<tr>
<td>15</td>
<td>1.020462</td>
<td>-17.474835</td>
<td>16</td>
<td>1.027567</td>
<td>-17.037637</td>
</tr>
<tr>
<td>17</td>
<td>1.018469</td>
<td>-17.393168</td>
<td>18</td>
<td>1.005346</td>
<td>-18.122725</td>
</tr>
<tr>
<td>19</td>
<td>1.000911</td>
<td>-18.305723</td>
<td>20</td>
<td>1.004929</td>
<td>-18.095308</td>
</tr>
<tr>
<td>21</td>
<td>1.008270</td>
<td>-17.689892</td>
<td>22</td>
<td>1.008590</td>
<td>-17.678446</td>
</tr>
<tr>
<td>23</td>
<td>1.003429</td>
<td>-17.894762</td>
<td>24</td>
<td>0.992111</td>
<td>-18.095032</td>
</tr>
<tr>
<td>25</td>
<td>0.978843</td>
<td>-17.724288</td>
<td>26</td>
<td>0.951628</td>
<td>-18.178112</td>
</tr>
<tr>
<td>27</td>
<td>0.984052</td>
<td>-17.197841</td>
<td>28</td>
<td>0.983836</td>
<td>-12.618487</td>
</tr>
<tr>
<td>29</td>
<td>0.951673</td>
<td>-18.624079</td>
<td>30</td>
<td>0.937921</td>
<td>-19.624272</td>
</tr>
</tbody>
</table>

Power flow in branch 6-4:

<table>
<thead>
<tr>
<th>Bus No. (From)</th>
<th>Bus No. (To)</th>
<th>( P_{6-12} + jQ_{6-12} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>4</td>
<td>-0.7390 + j0.0437</td>
</tr>
</tbody>
</table>
Then, the UPFC model is incorporated into the state estimation formulation. In this case, the system is underspecified and hence an extra equation is needed. This equation will be provided by the power flow measurement which will now be set equal to the desired value of the flow through the device in branch 6-4, which in this example is set equal to $-0.7 + j0.02$, leaving all the other conditions the same.

The estimated state variables in this case are:

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>V</th>
<th>$\theta$ (Degree)</th>
<th>Bus No.</th>
<th>V</th>
<th>$\theta$ (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.060000</td>
<td>0.000000</td>
<td>2</td>
<td>1.041838</td>
<td>-5.475726</td>
</tr>
<tr>
<td>3</td>
<td>1.016472</td>
<td>-7.995732</td>
<td>4</td>
<td>1.006776</td>
<td>-9.655479</td>
</tr>
<tr>
<td>5</td>
<td>1.011802</td>
<td>-14.350947</td>
<td>6</td>
<td>1.017156</td>
<td>-11.414607</td>
</tr>
<tr>
<td>7</td>
<td>1.007237</td>
<td>-13.143653</td>
<td>8</td>
<td>1.015091</td>
<td>-12.122612</td>
</tr>
<tr>
<td>9</td>
<td>1.070487</td>
<td>-14.395398</td>
<td>10</td>
<td>1.064048</td>
<td>-15.935696</td>
</tr>
<tr>
<td>11</td>
<td>1.115249</td>
<td>-14.395397</td>
<td>12</td>
<td>1.078960</td>
<td>-15.262148</td>
</tr>
<tr>
<td>13</td>
<td>1.109251</td>
<td>-15.262147</td>
<td>14</td>
<td>1.063893</td>
<td>-16.116744</td>
</tr>
<tr>
<td>15</td>
<td>1.058880</td>
<td>-16.191808</td>
<td>16</td>
<td>1.065156</td>
<td>-15.793472</td>
</tr>
<tr>
<td>17</td>
<td>1.059482</td>
<td>-16.101311</td>
<td>18</td>
<td>1.048793</td>
<td>-16.769330</td>
</tr>
<tr>
<td>19</td>
<td>1.045897</td>
<td>-16.928704</td>
<td>20</td>
<td>1.049670</td>
<td>-16.735522</td>
</tr>
<tr>
<td>21</td>
<td>1.051699</td>
<td>-16.364453</td>
<td>22</td>
<td>1.052165</td>
<td>-16.351227</td>
</tr>
<tr>
<td>23</td>
<td>1.047239</td>
<td>-16.554231</td>
<td>24</td>
<td>1.039953</td>
<td>-16.707064</td>
</tr>
<tr>
<td>25</td>
<td>1.031723</td>
<td>-16.253788</td>
<td>26</td>
<td>1.014302</td>
<td>-16.661620</td>
</tr>
<tr>
<td>27</td>
<td>1.035026</td>
<td>-15.723637</td>
<td>28</td>
<td>1.013930</td>
<td>-12.022441</td>
</tr>
<tr>
<td>29</td>
<td>1.015437</td>
<td>-16.925193</td>
<td>30</td>
<td>1.004106</td>
<td>-17.787005</td>
</tr>
</tbody>
</table>

The control variables and power of FACTS device are:

<table>
<thead>
<tr>
<th>$V_B$</th>
<th>$P_B$</th>
<th>$S_B$</th>
<th>$V_E$</th>
<th>$P_E$</th>
<th>$S_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1392$\angle$107.1456$^\circ$</td>
<td>0.0127</td>
<td>0.0976</td>
<td>0.9924$\angle$-11.5584$^\circ$</td>
<td>-0.0127</td>
<td>0.1235</td>
</tr>
</tbody>
</table>

Note again that, $P_B + P_E = 0$, and $V_B \leq 1.0$, $S_B \leq 1.0$, $V_E \leq 1.0$, $S_E \leq 1.0$, where all the constraints are met.
Now the power flow in branch 6-4 is \(-0.7001 + j0.0200\), which closely match the desired set values.

This example illustrates that by setting the control variables of UPFC to 
\[ V_B = 0.1392\angle 107.1456^\circ \quad \text{and} \quad V_E = 0.9924\angle -11.5584^\circ \], the power flow in branch 6-4 can be maintained at the desired amount.

3.3 Conclusion

This part of the report presents an algorithm for state estimation of power systems embedded with FACTS devices. While only the Unified Power Flow Controller (UPFC) is used in the development, other types of controllers can easily be integrated into the developed prototype with minor effort. This program may have dual purpose. It can be used to estimate the controller parameters along with system states during normal operation. It can also be used to determine the required controller settings in order to maintain a desired power flow through a given line. Simulation results on IEEE 14-bus and 30-bus test systems are shown in order to illustrate the proposed usage of the developed program.

REFERENCES


PART II: OPTIMAL METER PLACEMENT FOR MEASUREMENTS
LOSS AND BRANCH OUTAGES

I. INTRODUCTION

1.1 Introduction

Whether a new state estimator is put into service or an existing one is being upgraded, placing new meters for improving or maintaining reliability of the measurement system is of great concern. Determination of the best possible combination of meters for monitoring a given power system is referred to as the optimal meter placement problem. In choosing the types and locations of new measurements, there may be several different concerns, such as:

- Maintaining an observable network when one or more measurements are lost.
- Maintaining an observable network when one or more network branches are disconnected.
- Minimizing the cost of new metering

Our goal is to present a systematic procedure which can yield a measurement configuration that can withstand any one or more branch outages or loss of one or more measurements without losing network observability.

The paper [1] presented a topological method for single branch outages. The paper [2] proposed a unified approach, which generalized the meter placement problem formulation to simultaneously take into account both types of contingencies, namely loss of a branch or a measurement. The method is a numerical approach and can be implemented easily by modifying existing state estimation program. Furthermore, the total cost of adding measurements as well as the number of additional measurements are simultaneously minimized by an integer programming (IP) formulation. However, that method is only
valid for loss of single measurements and single branch outages. In reality, a given power system may be subjected to contingencies which include losses of multiple measurements and/or multiple branch outages. Moreover, the unified method of [2] provides a way to introduce candidates for a single contingency, which requires only one additional candidate measurement. If more than one candidate measurement is to be chosen for a contingency, then the IP problem needs to be reformulated so that proper IP constraints are used.

This part of the project addresses this need and improves the unified approach presented in [2] by extending it to the cases involving multiple measurement losses and multiple branch outages. The developed method is applied to several systems and results are presented.

1.2 Problem Statement

The performance of a state estimator includes considerations of accuracy, as well as reliability. A reliable state estimator should continue operating even under contingencies such as branch outages or temporary loss of measurements. On the other hand, budgetary constraints prohibit expansion of measurement systems for the sake of redundancy. Hence, we should look at an optimization problem where the number of meters should be kept at a minimum while ensuring network observability for a predetermined set of contingencies.

One indicator of observability is the column rank of the measurement Jacobian, $H$, whose column rank is not affected by the operating point, but essentially depends on the measurement configuration. Therefore, it is sufficient to evaluate $H$ at flat start in order to study the effects of branch outages of loss of measurements, on its rank.

Let the rows of Jacobian $H$ be ordered so that the first $m_e$ measurements are existing measurements. If the system is originally observable, the column rank of $H$ will be full, i.e. equal to $n$, i.e. the number of states. If $H$ is found rank deficient, then proper pseudo-measurements should be added to make the rank of $H$ full again. The
choice of these additional measurements must be optimal so that the overall cost of adding these measurements is a minimum.

The solution of this problem is obtained in two stages:

- One is “candidate measurements identification”, which is the selection of candidate measurement sets, each of which will make the system observable under a given contingency (loss of measurements and/or branch outages).
- The other is “optimal meter placement”, which is the choice of the optimal combination out of the selected candidate measurement sets in order to ensure the entire system observability under any one of the contingencies.

II. PROPOSED ALGORITHM

2.1 H matrix

H Matrix is the sub matrix representing the gradient of the real power measurements with respect to all bus phase angles, in the decoupled model. Let the rows of the H measurement Jacobian be ordered such that the existing measurements are listed first as shown below:

\[
H = \begin{bmatrix}
H_{\text{existing}} & m_e \text{ existing measurements} \\
\cdots & \\
H_c & m_c \text{ candidate measurements}
\end{bmatrix}
\]

Where, \( m = m_e + m_c \) is the total number of measurements that are either already existing (\( m_e \) measurements) or likely to be installed (\( m_c \) measurements).

- Loss of Measurements

For the loss of one existing measurement k, we can set all entries of the \( k^{th} \) row of the Jacobian H equal to zeros. If a contingency includes several measurement losses,
then we set all entries of corresponding rows equal to zeros and have modified H matrix like:

\[
H = \begin{bmatrix} H_{\text{existing}}^{\text{mod}} \\ \cdots \\ H_{e}^{\text{mod}} \end{bmatrix} m_c \text{ existing measurements} \\
\begin{bmatrix} \end{bmatrix} m_c \text{ candidate measurements}
\]

- **Loss of branches**

It is known that, network observability will be drastically affected by topology changes. Assuming that one contingency includes one or more branch outages, for each branch outage, say k-j branch is outage, some related elements of Jacobian are modified like:

\[
H_{ik}^{\text{mod}} = H_{ij}^{\text{mod}} = 0, \text{ if measurement } i \text{ is a line flow;}
\]

\[
H_{ij}^{\text{mod}} = 0, \ H_{ik}^{\text{mod}} = H_{ik} + H_{ij}, \text{ if measurements } i \text{ is an injection at bus } k.
\]

\[
H_{ik}^{\text{mod}} = 0, \ H_{ij}^{\text{mod}} = H_{ij} + H_{ik}, \text{ if measurements } i \text{ is an injection at bus } j.
\]

After modifying the related elements of Jacobian for all branch outages in that contingency, we have the measurement Jacobian modified as:

\[
H = \begin{bmatrix} H_{\text{existing}}^{\text{mod}} \\ \cdots \\ H_{e}^{\text{mod}} \end{bmatrix} m_c \text{ existing measurements} \\
\begin{bmatrix} \end{bmatrix} m_c \text{ candidate measurements}
\]

2.2 **Candidate measurements identification**

For each pre-determined contingency, we can obtain the modified Jacobian H matrix by the method mentioned above for loss of measurements and branches.

Triangular factorization of the modified H matrix with row pivoting within existing \( m_c \) measurements will yield the following factors:
\[ H = \begin{bmatrix} L_e \\ \vdots \\ M_e \\ \vdots \\ M_e \end{bmatrix} \begin{bmatrix} U_e \end{bmatrix} \quad (2.1) \]

Where, the sparse lower triangular matrix \( L_e \) and sparse rectangular matrix \( M_e \) are corresponding to the existing measurements, and sparse rectangular matrix \( M_e \) is corresponding the candidate measurements. \( U_e \) is sparse upper triangular matrices.

In carrying out the factorization procedure, row pivoting is restricted to the existing \( m_e \) measurements. If the rank of the sparse lower triangular matrix \( L_e \) is full, i.e. \( n \) for \( n+1 \) bus system, then the system is observable. If not, we have to select candidate rows from \( M_e \) to make the matrix rank full. Those selected candidate rows are corresponding to candidate measurements, which can be chosen to make the given system observable.

If the triangular factorization for the modified \( H \) matrix corresponding to one contingency still can be proceeded to \( n \)th row, which means that the rank of \( H \) matrix still is full after the contingency, we will say that the contingency does not affect the observability of the system so we do not need to search for any candidate measurements.

If the result of triangular factorization on the modified \( H \) matrix implies that the rank of the matrix is \( n-1 \), which means the contingency results in making the system unobservable, we can select candidates from the lower rectangular factor, which looks like below:
The measurements $j_1$, $j_2$ ... having nonzero in the $n$th column of the lower rectangular factor in the $M_c$ will be selected as candidates for that contingency.

More generally, if the factorization of the modified $H$ matrix for one contingency shows the rank is $n-k$; we will have the lower rectangular factor, which looks like below:

\[
\begin{bmatrix}
\times & 0 & \cdots & 0 \\
\times & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots \\
\times & \cdots & 0 \\
M_r & 0 & M_r \\
\end{bmatrix}
\]

(2.2)

For this case, additional $k$ measurements are needed, and we have to select candidates from $M_c$ to increase the rank.

For a given matrix $A$, the following properties are known to be true:

1. If $P$ is a nonsingular matrix and $PA = L$, then $\text{rank}(A) = \text{rank}(L)$

2. If $A = \begin{bmatrix} A_1 & 0 \\ A_2 & A_3 \end{bmatrix}$ then, $\text{rank}(A) = \text{rank}(A_1) + \text{rank}(A_3)$
Upon factorization of the $H$ matrix, those rows containing nonzero elements will be marked as possible candidates and denoted by the subscript “c”. Assuming that there are $C$ of such rows, any combination of $k$ rows among these $C$ nonzero rows, will yield the following:

$$\begin{bmatrix}
L_c & 0 \\
M_{cl} & M_{cr}
\end{bmatrix}$$

(2.4)

Where $M_{cl}$ and $M_{cr}$ have $k$ rows. If \( \text{rank}(M_{cr}) = k \), then based on the property 2 above, the overall rank of $H$ will be full. Otherwise, this set of possible candidates will be discarded from consideration since they will fail to render the system observable.

If we have $l$ nonzero rows in $M_c$, $(l \geq k)$, then we will have \( C^k_l \) sets of possible candidates. By the rank analysis of the sub matrix $M_{cr}$, we will know the number of candidates among \( C^k_l \) sets of possible candidates, which can be selected to make the system observable.

Admittedly, the approach described above has its drawbacks in terms of the CPU requirements, particularly if many additional candidate measurements are needed to make the system observable since we have to do search for all candidate measurement combinations. Clearly the fewer additional candidate measurements needed, the less complicated to find all sets of candidates. In practice though, there are very few cases that need more than five or more candidate measurements combined to make the system observable after a contingency, so this drawback is not considered to be of much practical significance.

2.3 Optimal Meter Placement

From the candidate selection procedure above, candidate measurements for each contingency can be obtained. The objective of the optimal selection procedure is to
minimize the overall cost of this measurement system upgrade while making sure that all contingencies are properly taken into account. Each candidate measurement will be assigned an installation cost.

In order to obtain the optimal meter placement for those pre-determined contingencies an Integer Programming (IP) Problem such as the one below is constructed:

\[
\text{to minimize } C^T \cdot X
\]

(2.5)

Where, C is a cost vector, and X is a binary candidate measurement status vector like:

\[
X(i) = \begin{cases} 
1 & \text{is measurement } i \text{ is selected} \\
0 & \text{otherwise}
\end{cases}
\]

(2.6)

The constraints of this IP problem will be:

- For the case that only one additional candidate measurement is necessary for the contingency,

\[
\sum_i x_i \geq 1 \text{ (ith measurement is the candidate for the contingency)}
\]

- For the case that additional two candidate measurements are necessary for the contingency,

\[
\sum_i x_{i_1} x_{i_2} \geq 1 \text{ (i}_1 \text{ \& i}_2 \text{ measurements are the candidates for the contingency)}
\]

- For the case that additional k candidate measurements are necessary for the contingency,

\[
\sum_{i_1} \prod_{k} x_{i_k} \geq 1 \text{ (Set i}_1 \text{...i}_k \text{ measurements are the candidates for the contingency)}
\]

The constraints in the IP problem ensures that each contingency is assigned candidate measurements whereas the objective function penalizes with respect to both the total cost as well as the number of selected candidates.
Solution of the IP problem described above yields measurements as the optimal choice that will ensure network observability under any pre-determined contingency at minimum cost.

2.4 Algorithm

The following algorithm is proposed for selecting candidates and determining optimal meter placement based on the above analysis:

**Step 1:** Form the measurements $H$ matrix, which include not only the existing measurements but also the non-existing measurements as the candidates.

**Step 2:** For a contingency modify the measurements $H$ matrix, then perform the triangular factorization with row pivoting and row exchange within existing measurement rows, and obtain the column rank of the matrix.

**Step 3:** Check if the column rank of the modified $H$ matrix is full. If yes, go to **Step 4**. If not, select the candidate measurements that can make the $H$ matrix full.

**Step 4:** Check if all the contingencies have been done. If not, go to **Step 2**. If yes, the IP problem is constructed based on all selected candidates.

**Step 5:** Yield the optimal meter placement, which ensures the entire system will remain observable under any one of the contingencies.

**Step 6:** Stop.
III. NUMERICAL EXAMPLES

2.1 6-bus system

The simple 6-bus system example with its measurement configuration shown in Figure 2.1 is considered to illustrate the proposed algorithm. All the branch impedances are set equal to j1.

![6-bus system example](image)

Figure 2.1. 6-bus system example

All the measurements shown in the above Figure 2.1 are considered as existing measurements, and all the injection measurements and flow measurements, which are not shown in Figure 2.1, are considered as candidate measurements.

Existing Measurements = [Injections: 1, 2, 6; Flows: 2-5, 3-4].

Candidate Measurements = [Injections: 3, 4, 5; Flows: 1-4, 1-6, 2-3, 4-6, 5-6].

The chosen installation cost vector $C^r$ corresponding to the candidate measurements is:

$$[1, 0.2, 0.4, 0.4, 0.5, 1, 1, 1]$$

Assume that $x_i$ is for the each candidate measurement.
In this system we suppose that the contingency list includes loss of each single existing measurement, outage of each single branch and two other contingencies:

Contingency 1: branch 4-6 outage and loss of injection measurements at buses 1 and 6;

Contingency 2: branch 2-3 outage and loss of injection measurement at bus 2.

The optimal meter placement algorithm should provide a set of additional candidate measurements, which will ensure network observability after any of contingencies in the list above.

Firstly we consider the optimal meter placement only for loss of single measurements and single branch outages like stated in [2], not consider the other two contingencies.

For each contingency (either loss of single measurement or single branch outage), at most one additional candidate measurement is needed to make the system observable.

By the method introduced in PART II, the candidate measurements, which is expressed as IP problem constraints, can be obtained for each existing measurement loss and each branch outage:

\[ \text{Inj.} 1 \text{ Loss: } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1 \]

\[ \text{Inj.} 2 \text{ Loss: } x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \geq 1 \]

\[ \text{Inj.} 6 \text{ Loss: } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1 \]

\[ \text{Flow2} - 5 \text{ Loss: } x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \geq 1 \]

\[ \text{Flow3} - 4 \text{ Loss: } x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1 \]

\[ \text{Branch} 2 - 3 \text{ Outage: } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1 \]

\[ \text{Branch} 2 - 5 \text{ Outage: } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1 \]
Branch 3 - 4 Outage: \( x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1 \)

The outages of single branches 1-4, 1-6, 4-6 and 5-6 will not affect the network observability so that no IP constraint will correspond to them. The solution of the integer-programming problem yields the injection measurement at bus-4 as the optimal choice that will ensure network observability under any single branch outage or loss of single measurement at minimum cost 0.2. This result is same as the result from the method in [2].

Secondly, we consider the other two contingencies, which include loss of several measurements and outage of several branches besides the contingencies of loss of single measurement and single branch outage. Besides the candidate measurements for the loss of single measurements and single branches, which are stated above, we also can obtain the candidate measurement sets for these two contingencies, which also are expressed as IP constraints:

Contingency 1: \( x_2 \cdot x_3 + x_2 \cdot x_4 + x_2 \cdot x_8 + x_3 \cdot x_4 + x_3 \cdot x_5 + x_4 \cdot x_3 + x_4 \cdot x_8 + x_5 \cdot x_8 \geq 1 \)

Contingency 2: \( x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1 \)

Obviously for the contingency 1, there are two additional candidate measurements needed to make the system observable. After considering these two contingencies, the IP solver shows that the optimal measurement set for this system is the injection measurements at bus 4 and 5 with the minimum installation cost 0.6. Hence, inclusion of these two additional measurements will maintain the system observable during any single line outage or loss of any single measurement and these two contingencies in the 6-bus system.
The IEEE-14 bus system with its measurement configuration shown in Figure 2.2 is also used to demonstrate the proposed method. All the measurements shown in the above Figure 2.2 are considered as existing measurements, and all the injection measurements and flow measurements, which are not shown in Figure 2.2 are considered as candidate measurements.

Existing Measurements = [Injections: 12, 13, 6, 11, 7, 8, 5, 9, 10; Flows: 9-14, 7-9, 4-7, 7-8, 1-2, 2-3].

Candidate Measurements = [Flows: 1-5, 2-4, 2-5, 3-4, 4-5, 4-9, 5-6, 6-11, 6-12, 6-13, 9-10, 10-11, 12-13, 13-14; Injections: 1, 2, 3, 4, 14].

The chosen installation cost vector $C^f$ corresponding to the candidate measurements is:

$[0.2, 1, 1, 1, 1, 0.5, 0.5, 1, 0.4, 1, 0.6, 1, 0.5, 1, 1, 0.3, 0.6, 0.9]$

Assume that $x_i$ is for the each candidate measurement.
Firstly we consider the optimal meter placement for loss of single measurements. As a result, the contingency list consists of loss of each existing measurement. For each contingency, at most one additional candidate measurement is needed to make the system observable. By the method introduced in PART II, the candidate measurements, which is expressed as IP problem constraints, can be obtained for each existing measurement loss:

\[ \text{Inj.12} : x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_{10} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1 \]

\[ \text{Inj.13} : x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_{10} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1 \]

\[ \text{Inj.6} : x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_{15} + x_{16} + x_{17} + x_{18} \geq 1 \]

\[ \text{Inj.11} : x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_8 + x_9 + x_{10} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1 \]

\[ \text{Inj.5} : x_1 + x_2 + x_3 + x_4 + x_{15} + x_{16} + x_{17} + x_{18} \geq 1 \]

\[ \text{Inj.9} : x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1 \]

\[ \text{Inj.10} : x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_8 + x_9 + x_{10} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1 \]

\[ \text{Flow9} - 14 : x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1 \]

\[ \text{Flow1} - 2 : x_1 + x_2 + x_3 + x_4 + x_{15} + x_{16} + x_{17} + x_{18} \geq 1 \]

\[ \text{Flow2} - 3 : x_4 + x_{16} + x_{17} + x_{18} \geq 1 \]

Finally solution of the integer-programming problem yields the injection measurement at bus-3 as the optimal choice that will ensure network observability under loss of any single measurement at minimum cost 0.3. However, since we exclude the redundant existing measurements in the candidate measurements, we
accordingly decrease the complexity of IP problem, which is important for the IP solver.

Secondly, we consider the contingencies including loss of several measurements and outage of single or several branches.

Contingency 1: loss of flow measurements in branch 1-2 and branch 2-3;
Contingency 2: loss of the injection measurement at bus 9, and loss of flow measurements in branch 9-14 and branch 9-7;
Contingency 3: loss of injection measurement at bus 7, and loss of flow measurements in branch 7-4 and branch 7-8;
Contingency 4: branch 10-11 outage;
Contingency 5: branch 9-10 outage.

Besides the candidate measurements for the loss of single measurements, we also can obtain the candidate measurement sets for these five pre-determined contingencies, which also are expressed as IP constraints:

\[ \text{Contingency 1: } x_2 \times x_4 + x_2 \times x_{16} + x_2 \times x_{17} + x_2 \times x_{18} + x_3 \times x_4 + x_3 \times x_{16} + \cdots \geq 1 \]

\[ \text{Contingency 2: } x_{11} \times x_1 + x_{11} \times x_2 + x_{11} \times x_3 + x_{11} \times x_4 + x_{11} \times x_5 + x_{11} \times x_7 + \cdots \geq 1 \]

\[ \text{Contingency 3: } x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_8 + \cdots \geq 1 \]

\[ \text{Contingency 4: } x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_8 + x_{10} + x_{13} + \cdots \geq 1 \]

\[ \text{Contingency 5: } x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_8 + x_{10} + x_{13} + \cdots \geq 1 \]

In each contingency, for space limitation not all sets of candidate measurements are listed above. After considering these five pre-determined contingencies, the IP solver
shows that the optimal measurement set for this system is to include the injection measurement at bus 3 and the flow measurement in branch 1-5. Hence, inclusion of these additional measurements will maintain the system observable during loss of any single measurement and these five pre-determined contingencies in the IEEE-14 bus system.

2.3 30-bus system

![Diagram of the IEEE-30 system with measurements set]

Existing Measurements = [Injections: 1, 2, 3, 5, 8, 9, 10, 12, 13, 15, 21, 24, 26, 27; Flows: 1-2, 1-3, 2-5, 2-6, 9-11, 12-13, 12-16, 14-15, 16-17, 15-18, 18-19, 10-21, 15-23, 22-24, 25-26, 25-27, 28-27, 29-30, 6-28].

Figure 2.3. IEEE-30 system with a measurements set
Candidate Measurements = [Injections: 4, 6, 7, 11, 14, 16, 17, 18, 19, 20, 22, 23, 25, 28, 29, 30; Flows: 2-4, 3-4, 4-6, 5-7, 6-7, 6-8, 6-9, 6-10, 9-10, 4-12, 12-14, 12-15, 19-20, 10-17, 10-20, 10-22, 21-22, 23-24, 24-25, 27-29, 27-30, 8-28].

The chosen installation cost vector $C^r$ corresponding to the candidate measurements is:

$[0.4, 0.4, 0.6, 0.8, 0.8, 0.4, 0.4, 0.4, 0.6, 1, 2, 1, 0.6, 0.6, 0.6, 0.2, 0.2, 2, 0.2, 2, 0.4, 1, 0.4, 1, 0.4, 0.4, 1, 0.8, 0.8, 0.6, 2, 0.4, 0.4, 0.2, 1, 0.2]$  

Assume that $x_i$ is for the each candidate measurement.

Similar to the previous two example systems, we consider loss of any single measurement at first. The corresponding IP constraints to loss of any single measurement are listed as follows:

* **Inj.5 Loss**: $x_2 + x_3 + x_{20} + x_{21} \geq 1$

* **Inj.8 Loss**: $x_2 + x_{14} + x_{22} + x_{38} \geq 1$

* **Inj.9 Loss**: $x_2 + x_9 + x_{10} + x_{23} + x_{25} + x_{29} + x_{31} \geq 1$

* **Inj.10 Loss**: $x_9 + x_{10} + x_{20} + x_{31} \geq 1$

* **Inj.12 Loss**: $x_1 + x_2 + x_7 + x_9 + x_{10} + x_{12} + x_{13} + x_{23} + x_{24} + x_{25} + x_{26} + x_{29} + x_{30} + x_{31} + x_{34} + x_{35} \geq 1$

* **Inj.15 Loss**: $x_1 + x_2 + x_5 + x_7 + x_9 + x_{10} + x_{12} + x_{13} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} + x_{28} + x_{29} + x_{30} + x_{31} + x_{34} + x_{35} \geq 1$

* **Inj.21 Loss**: $x_2 + x_7 + x_9 + x_{10} + x_{11} + x_{23} + x_{24} + x_{25} + x_{29} + x_{30} + x_{31} + x_{32} + x_{33} \geq 1$

* **Inj.24 Loss**: $x_2 + x_7 + x_9 + x_{10} + x_{12} + x_{13} + x_{23} + x_{24} + x_{25} + x_{29} + x_{30} + x_{31} + x_{34} + x_{35} \geq 1$
\[ \text{Inj.27 Loss: } x_{15} + x_{16} + x_{36} + x_{37} \geq 1 \]

\[ \text{Flow 9–11: } x_2 + x_4 + x_9 + x_{10} + x_{23} + x_{25} + x_{29} + x_{31} \geq 1 \]

\[ \text{Flow 12–16: } x_1 + x_2 + x_6 + x_7 + x_9 + x_{10} + x_{12} + x_{13} + x_{23} + x_{24} + x_{26} + x_{27} + x_{28} + x_{29} + x_{30} + x_{31} + x_{34} + x_{35} \geq 1 \]

\[ \text{Flow 14–15: } x_1 + x_2 + x_4 + x_7 + x_9 + x_{10} + x_{12} + x_{13} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} + x_{28} + x_{29} + x_{30} + x_{31} + x_{34} + x_{35} \geq 1 \]

\[ \text{Flow 16–17: } x_6 + x_7 + x_9 + x_{10} + x_{29} + x_{30} + x_{31} \geq 1 \]

\[ \text{Flow 15–18: } x_1 + x_2 + x_4 + x_7 + x_9 + x_{10} + x_{12} + x_{13} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} + x_{28} + x_{29} + x_{30} + x_{31} + x_{34} + x_{35} \geq 1 \]

\[ \text{Flow 18–19: } x_8 + x_9 + x_{10} + x_{29} \geq 1 \]

\[ \text{Flow 10–21: } x_2 + x_4 + x_9 + x_{10} + x_{11} + x_{23} + x_{24} + x_{25} + x_{29} + x_{30} + x_{31} + x_{32} + x_{33} \geq 1 \]

\[ \text{Flow 15–23: } x_1 + x_2 + x_3 + x_7 + x_9 + x_{10} + x_{12} + x_{13} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} + x_{28} + x_{29} + x_{30} + x_{31} + x_{34} + x_{35} \geq 1 \]

\[ \text{Flow 22–24: } x_2 + x_7 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{23} + x_{25} + x_{24} + x_{29} + x_{30} + x_{31} + x_{34} + x_{35} \geq 1 \]

\[ \text{Flow 25–27: } x_2 + x_7 + x_9 + x_{10} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{23} + x_{24} + x_{25} + x_{29} + x_{30} + x_{31} + x_{33} + x_{34} + x_{35} + x_{36} + x_{37} \geq 1 \]

\[ \text{Flow 28–27: } x_2 + x_7 + x_9 + x_{10} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{23} + x_{24} + x_{25} + x_{29} + x_{30} + x_{31} + x_{33} + x_{34} + x_{35} + x_{36} + x_{37} \geq 1 \]

\[ \text{Flow 29–30: } x_{15} + x_{16} + x_{36} + x_{37} \geq 1 \]

\[ \text{Flow 6–28: } x_2 + x_7 + x_9 + x_{10} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{23} + x_{24} + x_{25} + x_{29} + x_{30} + x_{31} + x_{33} + x_{34} + x_{35} + x_{36} + x_{38} \geq 1 \]
Compared with the A matrix in [2], obviously some injection measurement losses are not listed here, such as loss of injection measurements at bus 1, 2, 3, 13, 26. It is because loss of any of above five measurements will not affect the network observability and no extra measurement is needed.

Solving the IP problem as proposed, the optimal measurement set will be the injection measurements at buses 6, 19, and the flow measurement in branch 27-29 with minimum installation cost 1.2.

Next we include other 5 contingencies into the contingency list besides loss of any single measurement.

Contingency 1: loss of injection measurement at bus 1, and flow measurements in branches 1-2, 1-3;

Contingency 2: loss of injection measurement at bus 2, and flow measurements in branches 2-5, 2-6; branches 1-2 and 2-4 are outage;

Contingency 3: loss of injection measurement at bus 12, and flow measurements in branches 12-13, 12-16;

Contingency 4: loss of injection measurement at bus 26, and flow measurement in branch 25-26;

Contingency 5: loss of flow measurement in branch 29-30; branch 27-30 is outage;

As a result, for the given contingencies list, besides the candidate measurements for the loss of single measurements, we also can obtain the candidate measurement sets for these five contingencies, which also are expressed as IP constraints:

Contingency 1: \( x_1 + x_2 + x_3 + x_9 + x_{10} + x_{12} + x_{13} + x_{17} + x_{18} + x_{19} + x_{23} + x_{24} + x_{25} + x_{29} + x_{30} + x_{31} + x_{34} + x_{15} \geq 1 \)

Contingency 2: \( x_2 \cdot x_3 \cdot x_1 + x_2 \cdot x_3 \cdot x_7 + x_2 \cdot x_3 \cdot x_9 + x_2 \cdot x_3 \cdot x_{10} + x_2 \cdot x_3 \cdot x_{12} \cdot x_{13} \cdot x_{17} \cdot x_{23} \cdot x_{24} \cdot x_{25} \cdot x_{29} \cdot x_{30} + x_{31} + x_{34} + x_{15} \geq 1 \)

Contingency 3: \( x_6 \cdot x_7 + x_6 \cdot x_9 + x_6 \cdot x_{10} + x_6 \cdot x_{20} + x_6 \cdot x_{30} + \cdots \geq 1 \)
Contingency 4: \( x_{13} \geq 1 \)

Contingency 3: \( x_{15} + x_{16} \geq 1 \)

The IP solver shows that the optimal measurement set for this system is to include the injection measurements at buses 6, 19, 25, and 29, and the flow measurement in branch 5-7. Hence, inclusion of these additional measurements will maintain the system observable during loss of any single measurement and those five given contingencies in the IEEE-30 bus system.

2.4 57-bus system

![Figure 2.4. IEEE-57 system with a measurements set](image)

Existing Measurements = [Injections: 1, 3, 4, 5, 7, 8, 10, 11, 12, 13, 16, 17, 20, 21, 24, 29, 30, 31, 33, 34, 37, 38, 39, 44, 46, 48, 49, 52, 54, 55, 56, 57; Flows: 2-3, 4-5, 6-7,


The chosen installation cost vector $C^r$ corresponding to the candidate measurements is set equal to 0.1.

Assume that $x_i$ is for the each candidate measurement.

Firstly we consider the loss of any single measurement. After obtaining all the IP constraints corresponding to the loss of any single measurement, the optimal measurement set will be the injection measurements at buses 14, 32, with minimum installation cost of 0.2.

The corresponding IP constraints to loss of any single measurement are listed as follows:

**Inj.29 Loss**: $x_8 + x_9 + x_{13} + x_{14} + x_{15} + x_{47} + x_{49} + x_{53} + x_{54} \geq 1$

**Inj.7 Loss**: $x_8 + x_9 + x_{13} + x_{14} + x_{15} + x_{25} + x_{47}$

$+ x_{49} + x_{50} + x_{53} + x_{54} + x_{66} + x_{67} \geq 1$

**Inj.24 Loss**: $x_{10} + x_{14} + x_{15} + x_{48} + x_{53} + x_{54} \geq 1$

**Inj.30 Loss**: $x_{14} + x_{15} + x_{31} + x_{52} + x_{53} + x_{54} \geq 1$

**Inj.31 Loss**: $x_{14} + x_{15} + x_{32} + x_{53} + x_{54} \geq 1$
\textit{Inj.34} Loss : \( x_{14} + x_{15} + x_{33} + x_{54} \geq 1 \)

\textit{Inj.29} Loss : \( x_{8} + x_{9} + x_{13} + x_{14} + x_{15} + x_{47} + x_{49} + x_{53} + x_{54} \geq 1 \)

\textit{Inj.37} Loss : \( x_{14} + x_{15} + x_{16} + x_{17} + x_{53} + x_{54} + x_{55} + x_{57} \geq 1 \)

\textit{Inj.39} Loss : \( x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{33} + x_{54} + x_{55} + x_{56} + x_{57} + x_{72} + x_{73} + x_{74} + x_{75} \geq 1 \)

\textit{Inj.46} Loss : \( x_{4} + x_{22} + x_{60} + x_{61} \geq 1 \)

\textit{Inj.48} Loss : \( x_{4} + x_{22} + x_{60} + x_{61} + x_{62} \geq 1 \)

\textit{Inj.52} Loss : \( x_{8} + x_{9} + x_{13} + x_{14} + x_{15} + x_{25} + x_{47} + x_{49} + x_{53} + x_{54} + x_{66} + x_{67} \geq 1 \)

\textit{Inj.54} Loss : \( x_{8} + x_{9} + x_{13} + x_{14} + x_{15} + x_{25} + x_{47} + x_{49} + x_{53} + x_{54} + x_{66} + x_{67} + x_{68} \geq 1 \)

\textit{Inj.55} Loss : \( x_{8} + x_{9} + x_{13} + x_{14} + x_{15} + x_{25} + x_{47} + x_{49} + x_{53} + x_{54} + x_{66} + x_{67} + x_{68} + x_{69} \geq 1 \)

\textit{Inj.56} Loss : \( x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{33} + x_{54} + x_{55} + x_{56} + x_{57} + x_{72} + x_{73} + x_{74} + x_{75} \geq 1 \)

Flow 2 – 3 : \( x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{53} + x_{54} + x_{55} + x_{56} + x_{57} + x_{72} + x_{73} + x_{74} + x_{75} \geq 1 \)

Flow 24 – 26 : \( x_{8} + x_{9} + x_{10} + x_{11} + x_{14} + x_{15} + x_{47} + x_{48} + x_{53} + x_{54} \geq 1 \)

Flow 25 – 30 : \( x_{10} + x_{14} + x_{15} + x_{51} + x_{52} + x_{53} + x_{54} \geq 1 \)

Flow 41 – 43 : \( x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{53} + x_{54} + x_{55} + x_{56} + x_{57} + x_{58} + x_{70} + x_{72} + x_{73} + x_{74} + x_{75} \geq 1 \)

Flow 23 – 24 : \( x_{8} + x_{9} + x_{10} + x_{14} + x_{15} + x_{47} + x_{48} + x_{53} + x_{54} \geq 1 \)
Next, we include the other 5 contingencies into the contingency list besides the loss of any single measurement and outage of any single branch.

Contingency 1: loss of injection measurements at bus 4, and flow measurements in branches 4-5;

Contingency 2: loss of injection measurements at bus 24, and flow measurements in branches 23-24 and 24-26;

Contingency 3: loss of injection measurement at bus 48, and flow measurements in branches 48-49; branch 38-48 is outage;

Contingency 4: loss of injection measurement at bus 37, and flow measurement in branch 37-38; branch 36-37 is outage;

Contingency 5: loss of injection measurement at bus 3, loss of flow measurement in branches 2-3, 3-15; branch 3-4 is outage;

As a result, for the given contingency list, besides the candidate measurements for the loss of single measurements and single branches, we also can obtain the candidate
measurement sets for these five contingencies, which are expressed as the following IP constraints:

Contingency 1: \[ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{14} + x_{15} + x_{16} + \cdots \geq 1 \]

Contingency 2: \[ x_8 + x_9 + x_{10} + x_8 + x_9 + x_{14} + x_8 + x_9 + x_{15} + x_8 + x_9 + x_{48} + \cdots \geq 1 \]

Contingency 3: \[ x_4 + x_{22} + x_{60} + x_{61} + x_{62} \geq 1 \]

Contingency 4: \[ x_{14} + x_{12} + x_{16} + x_{17} + x_{33} + x_{54} + x_{27} \geq 1 \]

Contingency 5: \[ x_1 + x_3 + x_4 + x_5 + x_7 + x_{21} + x_i + x_{22} + \cdots \geq 1 \]

The IP solver shows that the optimal measurement set for this system is to include the injection measurements at buses 2, 22, 23, and the flow measurements in branches 34-35 and 46-47. Hence, inclusion of these additional measurements will maintain the system observable during any single line outage or loss of any single measurement and those five contingencies considered for the IEEE-57 bus system.

2.5 Conclusions

This part of the report presents several improvements to the unified measurement placement method by considering the loss of multiple measurements and/or multiple branch outages. Based on the modified measurement Jacobian H matrix for each contingency, a general candidate measurements selection method is introduced so that all candidates can be selected for loss of either single measurement and single branch or multiple measurements and multiple branches. Furthermore, the integer programming problem is extended to those cases where two or more candidates should be considered for placement due to a multiple contingency. Numerical examples verify the effectiveness of the proposed method for meter placement.
REFERENCES


1. Data Exchange on Distributed multi-utility operation [1, 3, 4]

- Criterion to quantify the performance of a measurement system is developed for data exchange design. A new concept of Bus Credibility Index (BCI) of Bus b is defined as the state estimation credibility probability on Bus b with respect to a specified system S.
- Based on BCI, an expert system is proposed to search for most beneficial data exchange, which consists of knowledge base and reasoning machine.
- Data exchange scheme is critical for its success: (Not all data exchanges are beneficial)
  - Properly selected data exchanges will enable the local distributed estimator perform as well as the one estimator for the whole system in both SE reliability and accuracy.
  - Poorly designed data exchanges, which does not follow our design principles, may be harmful to local estimators.
- Data exchange has an impact on new measurement design.
- Numerical tests on IEEE-14 bus system verify our above conclusions.

2. A Concurrent Textured Distributed State Estimation Algorithm

To avoid a huge cost of a new estimator for mega RTOs, we propose a cost effective distributed textured state estimator that maintains old state estimators with instrumentation or estimated data exchanges among neighboring entities.

Such an algorithm has the following features:

- Based on exchanging data with neighboring companies/ISOs/RTOs, textured overlapped areas become part of the process. With the developed textured decomposition method, bad data detection and identification ability is better than existing distributed state estimation algorithm, especially when bad data occur around the boundary of individual estimators.
- The distributed textured state estimator will be more reliable since one computer failure will not jeopardize the whole system estimation result.
- The distributed textured algorithm is non-recursive, asynchronous and avoids central controlling node. Therefore, it is fast and practical.
- Discrepancy on the boundary buses between different distributed estimators decreases and the result over whole grid become more consistent.
- When updating local estimation through estimated data exchanges, matrix modification techniques that utilize sparse techniques are developed to accelerate the computation speed.

Numerical tests on IEEE 14 bus system verify the efficiency and validity of the new approach.
References:


Conclusions

This project is concerned about the issues related to state estimation in the new power market operating environment. The first issue is the improvements needed in the measurement design in order to ensure reliable state estimation even under contingency conditions. This is addressed by developing a meter placement method which determines the least cost metering upgrade while accounting for contingencies. Another issue which is addressed is the incorporation of FACTS devices in to the state estimation formulation. This is accomplished by modifying the estimation problem formulation and including the operation and parameter limits of the FACTS devices. The issue of distributed multi-utility operation is addressed by developing a distributed textured decomposition state estimator and also by devising strategies for efficient measurement exchange between utilities.

The results of the project are implemented in form of prototype software and tested on typical power system configurations.